A Model of Campaign Spending in Repeated Elections

Jay Goodliffe*
Department of Political Science
Brigham Young University
Provo, Utah 84602-5545
goodliffe@byu.edu

April 2004

Abstract

I present a two-period election model of incumbent campaign spending where the challenger may learn about incumbent strength through incumbent behavior and performance, and decide whether to enter. The results indicate that challengers will focus on incumbent performance rather than behavior. These results are consistent with empirical findings on challenger entry.

*Prepared for delivery at the 2004 Annual Meetings of the Midwest Political Science Association, Chicago, Illinois, April 15-18, 2004. I am grateful to David Austen-Smith, Steve Brams, John Duggan, Mark Fey, Becky Morton, and John Patty for comments and suggestions on a previous version.

1 Introduction

Most formal models of incumbent behavior are principal-agent models with the median or representative voter as principal and the incumbent as agent. Incumbents are judged on their performance (competence or ideology) by voters. If the voters find the performance to be unsatisfactory, they pick a new incumbent, usually drawn randomly. These models study how the voter makes the retention decision in the presence of adverse selection (Rogoff and Sibert 1988, Rogoff 1990, Duggan 2000), moral hazard (Barro 1973, Ferejohn 1986, Austen-Smith and Banks 1989), or both (Banks and Sundaram 1993, 1998; Reed 1994).[1]

In presuming random choices, modelers suppress an important part of elections: the role of the challenger.[2] Challengers, particularly "high quality" challengers, are strategic in whether they choose to oppose an incumbent (Jacobson and Kernell 1983, Jacobson 1989, Banks and Kiewiet 1989, Canon 1990). Incumbents exert great efforts to appear strong to potential challengers, and potential challengers (and the press) constantly attempt to gauge the vulnerability (strength) of incumbents (Mayhew 1974, Fenno 1978, Stone and Maisel 2003, Herrnson 2004, Jacobson 2004). At the same time, incumbents are generally aware who their potential challengers are and their relative strength (Kazee 1980, 1983, 1994; Fowler and McClure 1989).

Some formal models have examined challenger entry. Banks and Kiewiet (1989) propose and test a model where high quality challengers run against each other for open seat races, and low quality challengers run against incumbents. Epstein and Zemsky (1995) create a model where preemptive fund raising *may* deter challenger entry. Dharmapala (2002) constructs a model of candidates, interest groups, and voters and finds that if fund-raising effectiveness is correlated with legislative effectiveness, then high fund-raising deters challengers. And Goodliffe (2003a, 2003b) develops models where incumbents' fund-raising and saving behavior can deter challengers from entering. While the evidence on preemptive fund-raising or war chests is mixed,[3] empirical studies agree that previous election performance has a strong effect on challenger entry decisions (e.g. Bond, Covington, and Fleisher 1985).

In this paper, I present a repeated election model where a challenger observes past election behavior and performance and decides whether to enter. To understand challenger entry and to make the model tractable, I de-emphasize the role of voters. Following Jacobson and Kernell (1983), I assume high quality challengers wish to run

[1] Persson and Tabellini (2000) call this approach "postelection politics," in constrast to preelection politics models, in which candidates can make binding electoral promises.

[2] Some studies of electoral competition endogenize challenger entry—most notably, the "citizen-candidate models" (Osborne and Slivinski 1996; Besley and Coate 1997), which are related to the following model—but most take the challenger as given.

[3] Studies that find support for deterrence effects include Goidel and Gross (1994), Hersch and MacDougall (1994), Box-Steffensmeier (1996), and Hogan (2001). Studies finding limited or no deterrence effects include Krasno and Green (1988), Squire (1991), Milyo (1998), Ansolabehere and Snyder (2000), and Goodliffe (2001, 2004).

against incumbents when the circumstances are favorable, i.e. when the incumbent is weak. Incumbent electoral performance is correlated (not perfectly) with incumbent strength; by observing previous vote totals, challengers can choose whether to enter. (Thus, incumbent "performance" is not ideology but electoral results.) Because incumbent strength is unobserved but incumbent actions are observed, this model resembles the models including adverse selection above. Because the incumbent's action is stochastically mapped into election performance, the model has characteristics similar to the models of Banks and Sundaram (1993, 1998).

In the equilibrium presented here, strong and weak incumbents have the same action in the initial election, and separate their actions in the latter election. Since the high quality challenger cannot learn about incumbent strength by observing initial incumbent actions, she observes the electoral performance of the incumbent. If the incumbent receives votes above a cutoff value, the high quality challenger does not enter. Since strong incumbents do (stochastically) better than weak incumbents, the high quality challenger will enter against weak incumbents more often than strong incumbents.

2 The Model

2.1 Decision Sequence

For simplicity, there are only two election cycles. The incumbent has an opportunity to raise and spend money in each election cycle. The time-line is as follows: a low quality challenger runs against the incumbent.[4] The incumbent then decides how much money he will raise and spend in this election. If he wins the election, the high quality challenger decides whether to enter the race or not, and then the incumbent again decides how much money to raise and spend in this second election cycle. (See Figure 1.)

2.2 Agents and Preferences

In its reduced form, there are two strategic agents in the model: an incumbent and a high quality challenger. There are two types of incumbents: strong and weak, $I \in \{S, W\}$. The probability of the incumbent being strong is $\pi = \Pr(I = S)$.Certainly, the strength of incumbents is not *completely* unknown, and the quality of challengers is not *completely* known. I justify the one-sided nature of uncertainty about candidate strength in two ways. First, even if challenger quality is unknown, or the incumbent does not know who the potential challengers are, the incumbent knows that he wants

[4] It is not essential that a low quality challenger run. It could be a high quality challenger instead (strictly, this would depend on π). But since the incumbent has no chance to act before the challenger must enter, the challenger cannot learn anything about the incumbent before she must enter.

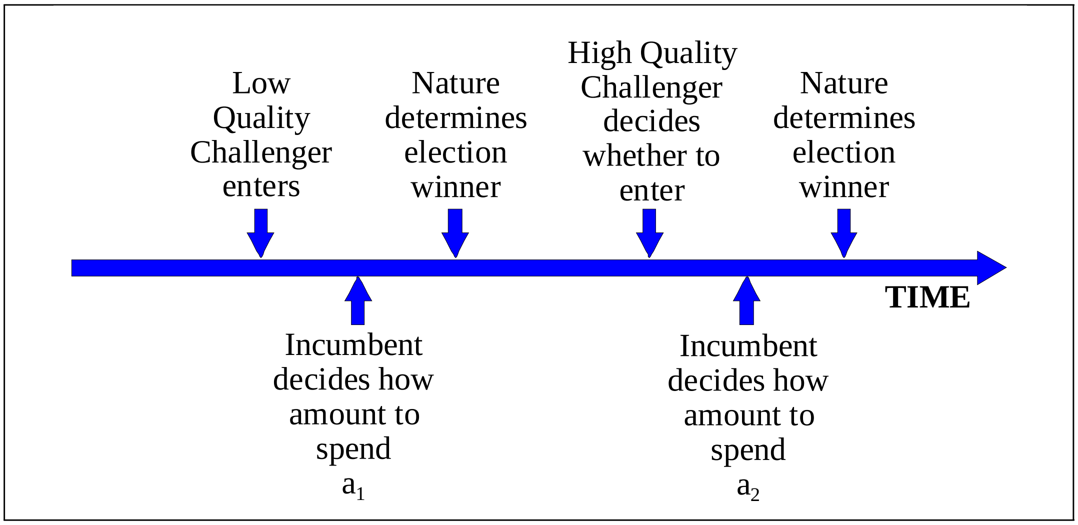

Figure 1: Decision Sequence

to deter the highest quality challengers (he would prefer to deter *all* challengers). Second, I am concerned with challenger entry—incumbents are (generally) already in the race. Decisions to challenge the incumbent usually hinge on challengers' perceptions of incumbent strength. I incorporate this dynamic by making incumbent strength (initially) unknown to potential challengers.

The incumbent's preferences are more complex. Let the incumbent's utility function (for one election) be given by:

$$U_{incumbent} = \begin{cases} b - C(\text{money raised}) & \text{if incumbent wins} \\ -C(\text{money raised}) & \text{else} \end{cases}$$

where b is the benefit of winning the election and $C(\cdot)$ is the cost of raising money. If the incumbent loses the first election, he gets 0 in the next election (i.e., does not run for office). Since the incumbent decides how much to raise (and spend) before the election, he maximizes his expected utility:[5]

$$EU_{incumbent} = \Pr\{\text{winning}\} \cdot b - C(\text{money raised}).$$

I normalize the benefit of winning, b, by setting $b = 1$. The probability of winning is determined by the incumbent's strength, the challenger's quality, and the action of the incumbent, in this case, the money raised and spent. Specifically, the incumbent's vote share is drawn from a continuous density $f(v|I, Q, a)$, where $v \in [0, 1]$ is the vote share, I is the incumbent strength, Q is the challenger quality, and a is the

[5]Since incumbents in this model seek to maximize their probability of reelection *while* minimizing their cost, the incumbents are empirically similar to those investigated by Milyo (2001).

incumbent's action. $F(v|I,Q,a)$ is the associated distribution (which is jointly continuous), and $w(I,Q,a) = 1 - F(.5|I,Q,a)$ is the expected probability of winning.[6] I assume that the set $\{v : f(v|I,Q,a) > 0\}$ is independent of a, so that the challenger cannot rule out a type of incumbent based on vote share. I also assume that $f(\cdot|\cdot,\cdot,\cdot)$ satisfies the Monotone Likelihood Ratio Property. Specifically, when $a > \hat{a}$, the ratio $f(v|I,Q,a)/f(v|I,Q,\hat{a})$ is strictly increasing in v. Similarly, when $I > \hat{I}$ $(S > W)$, the ratio $f(v|I,Q,a)/f(v|\hat{I},Q,a)$ is strictly increasing in v. And finally, when $Q > \hat{Q}$ $(H > L)$, the ratio $f(v|I,Q,a)/f(v|I,\hat{Q},a)$ is strictly decreasing in v. This means that $F(\cdot|I,Q,a)$ is ordered by first-order stochastic dominance. For example, when $a > \hat{a}$, $F(v|I,Q,a) < F(v|I,Q,\hat{a})$ and $w(I,Q,a) > w(I,Q,\hat{a})$ (or $\frac{\partial w}{\partial a}(I,Q,a) > 0$). In other words, raising and spending more money yields a higher expected probability of winning; strong incumbents have a higher expected probability of winning than weak incumbents; and high quality challengers have a greater chance of defeating the incumbent than low quality challengers, ceteris paribus.

In addition, $\frac{\partial^2 w}{\partial a^2}(I,Q,a) < 0$. That is, spending more money increases the expected probability of winning, but the marginal value of spending money is decreasing. I further assume that for any given spending, an incumbent facing a high quality challenger receives higher returns to spending than an incumbent facing a low quality challenger: $\frac{\partial w}{\partial a}(I,H,a) > \frac{\partial w}{\partial a}(I,L,a)$ for all I and a (or $\frac{\partial^2 w}{\partial a \partial Q}(I,Q,a) > 0$). Similarly, assume that the returns to spending by a strong incumbent is less than the returns to spending by a weak incumbent: $\frac{\partial w}{\partial a}(S,Q,a) < \frac{\partial w}{\partial a}(W,Q,a)$ for all Q and a (or $\frac{\partial^2 w}{\partial a \partial I}(I,Q,a) < 0$). This means that the money raised (and spent) against a high quality challenger if there were only one election is greater than the money raised (and spent) against a low quality challenger; and that the money raised (and spent) by a strong incumbent if there were only one election is less than the money raised (and spent) by a weak incumbent.[7] If incumbent strength were known to all challengers, when one aggregates across races with different incumbent strengths and challenger qualities, it will appear that the more an incumbent spends, the worse he does in the election. I also assume Inada-type conditions so that I obtain an interior solution.

I assume that $C(a)$ is a twice differentiable function, where a is the amount of money raised. I further assume $C'(a) > 0$ and $C''(a) > 0$, so that both the cost and marginal cost of raising money increases as the amount of money raised increases.

[6] The results are qualitatively similar if the incumbent maximizes vote share: $\bar{v}(I,Q,a) = \int v f(v|I,Q,a)dv$.

[7] Thus, I and Q have similar (but opposite) effects on F. It is somewhat counter-intuitive to think of strong incumbents spending less than weak incumbents, ceteris paribus. This is a result of treating incumbent strength and challenger quality as similar concepts. As an alternative formulation, one could use $F(v|I-Q,a)$.

Like the win probability function, I assume Inada-type conditions. Note that the cost function does not depend on the strength of the incumbent or the quality of challenger running. Also assume that there exists some $\hat{a}$ such that $w(I, Q, \hat{a}) > C(\hat{a})$, for all I and Q, else the incumbent would not run for re-election.

Finally, assume that the utility gain to a weak incumbent from deterring a high quality challenger is greater than the utility gain for a strong incumbent.[8] In contrast to other principal-agent models, these restrictions on f and C induce a utility function that is *submodular* in incumbent strength and spending (Milgrom and Shannon 1984).[9] What this means substantively is that weak incumbents get more out of monies spent than strong incumbents. Figure 2 displays a cost function and sample win probability functions for an incumbent facing high quality or low quality challengers. Let a_L be the a that solves $\frac{\partial w}{\partial a}(I, L, a) = C'(a)$ for a given I, and similarly for a_H. The figure also shows a_L and a_H in the form of vertical lines. Incumbents prefer to run against low quality challengers.

The incumbent may not borrow money, and must spend the money on hand.[10]

There are two challengers: high and low quality, $Q \in \{H, L\}$. To focus attention on the incumbent's decision, the high quality challenger merely decides whether to enter the race or not. Like the incumbent's utility function, the high quality challenger receives a benefit from winning and a cost of running. The probability of winning is the complement of the incumbent's probability of winning: $1 - w(I, H, a) = F(.5|I, H, a)$. Following Jacobson and Kernell (1983), higher quality challengers have more to lose when running (often, they currently hold office, which they would lose by running).[11] Thus, the utility function for the high quality challenger is:

$$EU_{HQ\ challenger} = [1 - w(I, H, a)] \cdot b - c^H,$$

[8]In other words, denoting $u(I, Q, a^{I,Q}) = w(I, Q, a^{I,Q}) - C(a^{I,Q})$,

$$u(W, L, a^{W,L}) - u(W, H, a^{W,H}) > u(S, L, a^{S,L}) - u(S, H, a^{S,H}).$$

[9]A utility function is submodular in a and I if whenever $a > \hat{a}$ and $I > \hat{I}$,

$$u(a, I) - u(a, \hat{I}) < u(\hat{a}, I) - u(\hat{a}, \hat{I}).$$

Let $u(a, I) = \bar{v}(I, Q, a) - C(a)$, $a = a_W$, $\hat{a} = a_S$, $I = S$, and $\hat{I} = W$. Substituting in and canceling out the cost functions,

$$w(S, Q, a_W) - w(W, Q, a_W) < w(S, Q, a_S) - w(W, Q, a_S).$$

This inequality is satisfied given the restrictions on f and C.

[10]See Goodliffe (2002, 2003a, 2003b) for models where the incumbent is allowed to save money from one election to the next.

[11]There is a sizable literature assessing challenger quality. See Squire (1995) for a survey. For this paper, I do not worry what constitutes a high quality challenger; I merely assume that there is a difference between high and low quality challengers.

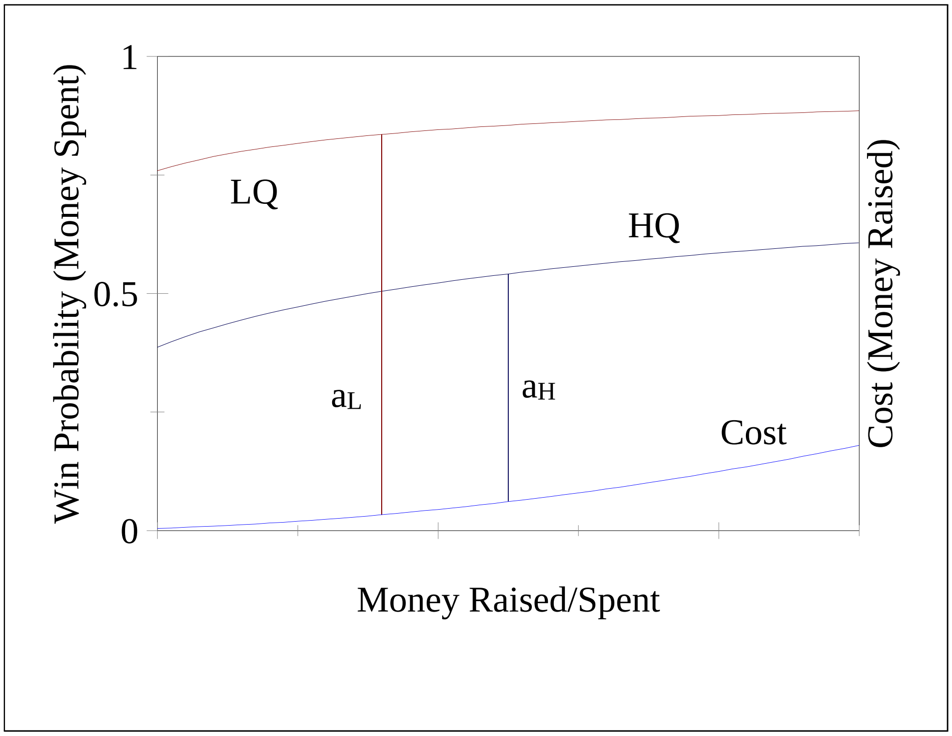

Figure 2: Expected Win Probabilities for Challenger Qualities

where b is the benefit of office (normalized to 1), and c^H is the high quality challenger's cost of running for office.[12] From the assumptions above, the high quality challenger prefers to run against the weak incumbent, given she enters the race. The high quality challenger can also choose not to enter the race, in which case she receives a payoff of 0. Let $a_2^{I,Q}$ be the a that solves $\frac{\partial w}{\partial a}(I, Q, a) = C'(a)$. I assume that $1 - w(S, H, a_2^{S,H}) < c^H < 1 - w(W, H, a_2^{W,H})$. This implies that if incumbent strength is known, the high quality challenger will not run against the strong incumbent, but will run against a weak incumbent.

There is also a low quality challenger in the model, whose preferences are such that she will always run if the high quality challenger does not run (and will not run if the high quality challenger does run[13]). This is justified by the fact that low quality challengers are often motivated by different factors than high quality challengers

[12] To concentrate on the incumbent's fund-raising, spending and saving decisions, the challenger does not raise or spend money. The win and cost functions can be thought of as reduced-form expressions of a game where both candidates raise and spend money.

[13] There are cases in which both a high quality challenger and low quality challenger run against each other in the primary. Banks and Kiewiet (1989) and Canon (1990) find that the high quality challenger usually wins the primary.

(Canon 1990). The utility function for such a challenger is:

$$EU_{LQ\ challenger} = [1 - w(I, L, a)] \cdot b - c^L,$$

where c^L is zero or so low that the low quality challenger will always enter if the high quality challenger does not. This is justified by the fact that low quality challengers are often motivated by different factors than high quality challengers (Canon 1990).

I denote the probability that a high [low] quality challenger runs against the incumbent as η $[1 - \eta]$, where $0 \leq \eta \leq 1$. Denoting the election with subscripts and the quality of challenger the incumbent faces with superscripts, the incumbent's expected utility function over two election cycles is:

$$\begin{array}{l} \eta_1 \left\{ -C(a_1^H) + w(I, H, a_1^H) \left[\begin{array}{c} 1 + \eta_2 \left\langle w(I, H, a_2^H) - C(a_2^H) \right\rangle \\ +(1 - \eta_2) \left\langle w(I, L, a_2^L) - C(a_2^L) \right\rangle \end{array} \right] \right\} \\ +(1 - \eta_1) \left\{ -C(a_1^L) + w(I, L, a_1^L) \left[\begin{array}{c} 1 + \eta_2 \left\langle w(I, H, a_2^H) - C(a_2^H) \right\rangle \\ +(1 - \eta_2) \left\langle w(I, L, a_2^L) - C(a_2^L) \right\rangle \end{array} \right] \right\} \end{array}$$

In the second period, if the high quality challenger decides not to enter the race, a low quality challenger will run against the incumbent.[14] Note that there is a built-in discount factor for incumbents: the expected probability of winning the first election. To summarize preferences:

$$EU_{HQ\ challenger} = \begin{cases} [1 - w(I, H, a)] - c^H & \text{enter against incumbent} \\ 0 & \text{not enter} \end{cases}$$

$$EU_{incumbent} = \left\{ \begin{array}{l} \eta_1 \left\{ -C(a_1^H) + w(I, H, a_1^H) \left[\begin{array}{c} 1 + \eta_2 \left\langle w(I, H, a_2^H) - C(a_2^H) \right\rangle \\ +(1 - \eta_2) \left\langle w(I, L, a_2^L) - C(a_2^L) \right\rangle \end{array} \right] \right\} \\ +(1 - \eta_1) \left\{ -C(a_1^L) + w(I, L, a_1^L) \left[\begin{array}{c} 1 + \eta_2 \left\langle w(I, H, a_2^H) - C(a_2^H) \right\rangle \\ +(1 - \eta_2) \left\langle w(I, L, a_2^L) - C(a_2^L) \right\rangle \end{array} \right] \right\} \end{array} \right.$$

2.3 Information

The decision sequence is common knowledge. Similarly, the challenger's and incumbent's preferences are common knowledge up to the value of I. The challenger has a common knowledge prior belief ($\pi = \Pr(I = S)$) over the incumbent's type, while the incumbent knows his type.

2.4 Strategies

Without loss of generality, I restrict attention throughout to pure strategies.

The incumbent's *first-election spending* strategy is a map:

$$\mu : I \to \mathbb{R}_+$$

[14] In this model η_1 is assumed, and η_2 is determined endogenously (by the high quality challenger).

Thus, for any incumbent I, $\mu(I) = a_1$.

After observing the incumbent's first election spending and his vote share, the challenger forms a belief about the type of incumbent that may have generated a_1 and v_1. Let $\rho(I|a_1, v_1)$ be that probability. The challenger's *entrance* strategy is a map:

$$\sigma : \mathbb{R}^2_+ \rightarrow \{\text{enter, not enter}\}$$

Thus, for the challenger (whose quality, Q, is common knowledge), $\sigma(a_1, v_1)$ is the challenger's (non-)entrance, having observed the incumbent's spending in the first election and the vote share, given a low quality challenger entered in the first election.

After the challenger has decided whether to enter, the incumbent decides how much to spend in the last election. The incumbent's *second-election spending* strategy is a map:

$$\gamma : I \times \{H, L\} \rightarrow \mathbb{R}_+$$

where $\gamma(I, Q) = a_2$ is the amount spent in the second (and last) election given the *action* of the challenger in the second election. Thus, a set of strategies is $(\sigma^*, \mu^*, \gamma^*)$.

Since the challenger has incomplete information about the preferences of the incumbent, an appropriate equilibrium concept for this game is *sequential equilibrium* (Kreps and Wilson 1982). Loosely, incumbents and challengers maximize their expected payoffs at every point in time, where the expectation is with respect to the challenger's beliefs, updated by Bayes' rule when possible.

3 Equilibrium

In this model, all incumbents separate in their second period (optimal) spending, as there no further actions to influence. However, incumbents may pool or separate in their first period spending.

3.1 Pooling Equilibrium

I characterize the pooling equilibrium, where the weak incumbent imitates the strong incumbent.

Proposition 1 *(Pooling) There exists an equilibrium $(\sigma^*, \mu^*, \gamma^*)$ in which:*

$$\begin{aligned} \sigma^*(a_1, v_1) =& \ \{\text{not enter}\} && \forall v \geq \hat{v} \\ \sigma^*(a_1, v_1) =& \ \{\text{enter}\} && \forall v < \hat{v} \\ \mu^*(I) =& \ a_1^{S,L} && \forall I \\ \gamma^*(I, Q) =& \ a_2^{I,Q} && \end{aligned}$$

where $a_2^{I,Q}$ is the amount raised and spent by incumbent I against challenger Q.

I construct strategies by solving the game backwards.

3.1.1 Incumbent Second-Election Spending

The incumbent—who cannot run again, knows his type and knows the quality of the challenger—simply maximizes his utility and chooses $a = a_2^{I,Q}$ such that $\frac{\partial w}{\partial a}(I, Q, a) = C'(a)$.

3.1.2 Challenger Entry

Having observed incumbent spending and vote share in the first election, the challenger updates her beliefs, and decides whether to enter. The challenger enters if $\rho(S|a_1, v_1)\left[1 - w(W, H, a_2^{W,H})\right] + (1 - \rho(S|a_1, v_1))\left[1 - w(W, H, a_2^{W,H})\right] - c^H > 0$. Because both types of incumbents have the same action (amount of spending), the challenger updates her beliefs by using Bayes' rule on vote share: $\rho(S|a_1, v_1) = \frac{\pi f(v_1|S, L, a_1)}{\pi f(v_1|S, L, a_1) + (1-\pi) f(v_1|W, L, a_1)}$. Out-of-equilibrium beliefs are set such that defections are assigned to the weak incumbent. Given a_1 and π, if $v_1 = \hat{v}_1$, then enough weight is placed on strong incumbents that the challenger is indifferent between entering and not entering.

3.1.3 Incumbent First-Election Spending

The strong incumbent solves the following problem:

$$\max_{a_1^{S,L}} -C(a_1^{S,L}) + w(S, L, a_1^{S,L}) \left\{ 1 + \left\langle \begin{array}{c} \left[1 - F(\hat{v}|S, L, a_1^{S,L})\right]\left[w(S, L, a_2^{S,L}) - C(a_2^{S,L})\right] \\ + \left[F(\hat{v}|S, L, a_1^{S,L})\right]\left[w(S, H, a_2^{S,H}) - C(a_2^{S,H})\right] \end{array} \right\rangle \right\}$$

where $\left[1 - F(\hat{v}|S, L, a_1^{S,L})\right]$ is the probability of obtaining a vote share greater than $\hat{v}$. Because the incumbent is maximizing money spent for the marginal benefit of this election and the (probability) of the next election, he spends more in the first election than in the second: $a_1^{S,L} > a_2^{S,L}$, given the same challenger quality.

The weak incumbent simply chooses the same action as the strong incumbent. Interestingly, this means that he actually spends less than he would if his type were known with certainty (and he did not attempt to imitate). Although the weak incumbent does worse in the first election than he would if he were to choose this certainty amount, he makes up for it by reducing the probability that a high quality challenger will run against him in the second election. Specifically, the weak incumbent imitates the strong incumbent if

$$\begin{aligned} & -C(a_1^{S,L}) + w(W, L, a_1^{S,L}) \left\{ 1 + \left\langle \begin{array}{c} \left[1 - F(\hat{v}|W, L, a_1^{S,L})\right]\left[w(W, L, a_2^{W,L}) - C(a_2^{W,L})\right] \\ + \left[F(\hat{v}|W, L, a_1^{S,L})\right]\left[w(W, H, a_2^{W,H}) - C(a_2^{W,H})\right] \end{array} \right\rangle \right\} \\ \geq \; & -C(a_1^{W,L}) + w(W, L, a_1^{W,L}) \left\{ 1 + w(W, H, a_2^{W,H}) - C(a_2^{W,H}) \right\} \end{aligned}$$

Given the assumptions on C and F, this inequality is satisfied.

Out-of-equilibrium beliefs are set such that a challenger who observes an out-of-equilibrium spending amount assumes that it is the weak incumbent. Since defecting from the equilibrium makes the challenger enter with certainty, neither type of incumbent has incentive to deviate.

3.2 Separating Equilibria

In a separating equilibrium, strong incumbents take different actions than weak incumbents in the first election. Since the incumbent reveals his strength, the challenger enters against weak incumbents and does not enter against strong incumbents. I show that there are no separating equilibria of this form.

Proposition 2 *There exists no equilibrium in which μ^* is separating on a_1.*

Suppose not. In a separating equilibrium, each type incumbent has no incentive to imitate the other. By incentive compatibility:

$$\begin{aligned}
& -C(a_1^{S,L}) + w(S,L,a_1^{S,L}) + w(S,L,a_1^{S,L})\left\{-C(a_2^{S,L}) + w(S,L,a_2^{S,L})\right\} \\
\geq\ & -C(a_1^{W,L}) + w(S,L,a_1^{W,L}) + w(S,L,a_1^{W,L})\left\{-C(a_2^{S,H}) + w(S,H,a_2^{S,H})\right\} \\
& -C(a_1^{W,L}) + w(W,L,a_1^{W,L}) + w(W,L,a_1^{W,L})\left\{-C(a_2^{W,H}) + w(W,H,a_2^{W,H})\right\} \\
\geq\ & -C(a_1^{S,L}) + w(W,L,a_1^{S,L}) + w(W,L,a_1^{S,L})\left\{-C(a_2^{W,L}) + w(W,L,a_2^{W,L})\right\}.
\end{aligned}$$

Combining and simplifying the two equations,

$$\begin{aligned}
& w(S,L,a_1^{S,L})\left\{1 - C(a_2^{S,L}) + w(S,L,a_2^{S,L})\right\} \\
& -w(S,L,a_1^{W,L})\left\{1 - C(a_2^{S,H}) + w(S,H,a_2^{S,H})\right\} \\
\geq\ & w(W,L,a_1^{S,L})\left\{1 - C(a_2^{W,L}) + w(W,L,a_2^{W,L})\right\} \\
& -w(W,L,a_1^{W,L})\left\{1 - C(a_2^{W,H}) + w(W,H,a_2^{W,H})\right\}.
\end{aligned}$$

By the assumptions on C and F,[15]

$$\begin{aligned}
& \left\{-C(a_2^{W,L}) + w(W,L,a_2^{W,L})\right\} - \left\{-C(a_2^{W,H}) + w(W,H,a_2^{W,H})\right\} \\
>\ & \left\{-C(a_2^{S,L}) + w(S,L,a_2^{S,L})\right\} - \left\{-C(a_2^{S,H}) + w(S,H,a_2^{S,H})\right\}
\end{aligned}$$

[15] See footnote 8.

and

$$w(W, L, a_1^{S,L}) - w(W, L, a_1^{W,L}) > w(S, L, a_1^{S,L}) - w(S, L, a_1^{W,L}).$$

However, combining these two strict inequalities leads to the inequality:

$$\begin{aligned} & w(S, L, a_1^{S,L}) \left\{1 - C(a_2^{S,L}) + w(S, L, a_2^{S,L})\right\} \\ & -w(S, L, a_1^{W,L}) \left\{1 - C(a_2^{S,H}) + w(S, H, a_2^{S,H})\right\} \\ < \; & w(W, L, a_1^{S,L}) \left\{1 - C(a_2^{W,L}) + w(W, L, a_2^{W,L})\right\} \\ & -w(W, L, a_1^{W,L}) \left\{1 - C(a_2^{W,H}) + w(W, H, a_2^{W,H})\right\}, \end{aligned}$$

in direct contradiction to the incentive compatibility conditions. Therefore, there can be no separating equilibria.

4 Discussion

The equilibrium presented has several interesting properties which provide alternative explanations for empirical phenomena. First, all incumbents act the same in the first election, and only act differently (reveal their types) in the second election. However, strong and weak incumbents get different results in vote share (stochastically). Weak incumbents will lose more often (because $w(S, L, a_1^{S,L}) > w(W, L, a_1^{S,L})$), and high quality challengers will enter against weak incumbents more often (because $\left[1 - F(\hat{v}|S, L, a_1^{S,L})\right] > \left[1 - F(\hat{v}|W, L, a_1^{S,L})\right]$). Empirically, one should observe first-term (or relatively new) incumbents acting similarly. Only as the incumbent gains experience (and incumbent strength becomes known) should we see divergence in behavior.

Second, controlling for challenger quality, incumbents spend more in the first election than the second election. This is because the incumbent is equating the marginal cost with the marginal benefit of current *and* future elections. The conventional wisdom is that first-term incumbents spend a lot of money on their next election because they are vulnerable. However, the explanation here is that first-term incumbents are spending more because they are considering the present and future elections.

Third, high quality challengers will run against both strong and weak incumbents (although more frequently against the latter). This is because even strong incumbents can have low vote share, although probabilistically it will happen less often than for weak incumbents. Empirically, we observe that high quality challengers win more often than low quality challengers (Jacobson 1989). Two factors in this model explain the result. High quality challengers do better than low quality challengers (by assumption in this model); and high quality challengers run more often against weak incumbents than strong incumbents (derived from the equilibrium of this model).

If current empirical work does not control for the strength of incumbents, then the effect of being a high quality challenger *per se* is overstated.

Finally, as incumbent strength becomes known, it will appear that the more the incumbent spends, the worse he appears to do. This is consistent with the first generation of incumbent spending papers which found a negative return to spending. Even controlling for endogeneity with sophisticated methods, there is no consensus as to the direction (zero or positive) and magnitude of the effect of incumbent spending.

It is interesting to note the difference between this equilibrium, and those found in Banks and Sundaram (1993, 1998). In Banks and Sundaram's models, the different incumbents separate in their actions (although they are not observed). There are two reasons why this may be different. First, actions are observed in this model, and not in Banks and Sundaram. Second, in Banks and Sundaram (1998), the incumbent's (agent's) utility is supermodular, whereas in this model, the utility function $(w - C)$ is submodular.

5 Future Work

The purpose of this paper was to introduce a model where the challenger attempts to learn about the incumbent through incumbent behavior and incumbent performance. In the equilibrium presented, the challenger was only able to learn about incumbent strength through incumbent performance, not behavior. This result comports with the empirical finding that challengers respond to incumbent performance in deciding whether to challenge the incumbent.

Future work on this model will include investigating and characterizing other equilibria besides the one presented here (and then applying appropriate refinements). The model will also be extended to allow multiple (possibly a continuum of) types of incumbents and perhaps challengers. In addition, incumbent strength could be operationalized by the marginal cost of fund-raising. Further, instead of observing incumbent spending, the model could incorporate moral hazard by making the incumbent's effort at fund-raising, which is not directly observed, as the relevant action. Finally, it would also be instructive to extend this to an infinitely repeated election model, instead of the two election model presented here.

References

[1] Austen-Smith, David, and Jeffrey S. Banks. 1989. "Electoral Accountability and Incumbency." In *Models of Strategic Choice in Politics*, ed. Peter Ordeshook. Ann Arbor, Michigan: University of Michigan Press.

[2] Ansolabehere, Stephen, and James M. Snyder, Jr. 2000. "Campaign War Chests in Congressional Elections." *Business and Politics* 2:9-33.

[3] Banks, Jeffrey S., and D. Roderick Kiewiet. 1989. "Explaining Patterns of Candidate Competition in Congressional Elections." *American Journal of Political Science* 33:997-1015.

[4] Banks, Jeffrey S., and Rangarajan K. Sundaram. 1993. "Adverse Selection and Moral Hazard in a Repeated Elections Model." In *Political Economy: Institutions, Competition, and Representation*, eds. William A. Barnett, Melvin J. Hinich, and Norman J. Schofield. New York, New York: Cambridge University Press.

[5] Banks, Jeffrey S., and Rangarajan K. Sundaram. 1998. "Optimal Retention in Agency Problems." *Journal of Economic Theory* 82:293-323.

[6] Barro, Robert J. 1973. "The Control of Politicians: An Economic Model." *Public Choice* 14:19-42.

[7] Besley, Timothy, and Stephen Coate. 1997. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics* 112:85-114.

[8] Bond, Jon R., Cary Covington, and Richard Fleisher. 1985. "Explaining Challenger Quality in Congressional Elections." *Journal of Politics* 47:510-529.

[9] Box-Steffensmeier, Janet M. 1996. "A Dynamic Analysis of the Role of War Chests in Campaign Strategy." *American Journal of Political Science* 40:352-371.

[10] Canon, David T. 1990. *Actors, Athletes, and Astronauts: Political Amateurs in the United States Congress*. Chicago: University of Chicago Press.

[11] Dharmapala, Dhammika. 2002. "Campaign War Chests, Entry Deterrence, and Voter Rationality." *Economics and Politics* 14(3):325-350.

[12] Duggan, John. 2000. "Repeated Elections with Asymmetric Information." *Economics and Politics* 12:109-135.

[13] Epstein, David, and Peter Zemsky. 1995. "Money Talks: Deterring Quality Challengers in Congressional Elections." *American Political Science Review* 89:295-308.

[14] Fenno, Richard F., Jr. 1978. *Home Style: House Members in Their Districts*. Boston: Little, Brown.

[15] Ferejohn, John A. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50:5-25.

[16] Fowler, Linda L., and Robert D. McClure. 1989. *Political Ambition: Who Decides to Run for Congress*. New Haven: Yale University Press.

[17] Goidel, Robert K., and Donald A. Gross. 1994. "A Systems Approach to Campaign Finance in U.S. House Elections." *American Politics Quarterly* 22:125-153.

[18] Goodliffe, Jay. 2001. "The Effect of War Chests on Challenger Entry in U.S. House Elections." *American Journal of Political Science* 45:830-844.

[19] Goodliffe, Jay. 2002. "War Chests as Precautionary Savings." Working Paper.

[20] Goodliffe, Jay. 2003a. "War Chests and Information." Working Paper.

[21] Goodliffe, Jay. 2003b. "When Do War Chests Deter?" Working Paper.

[22] Goodliffe, Jay. 2004. "Campaign War Chests and Challenger Quality in Senate Elections." Working Paper.

[23] Herrnson, Paul S. 2004. *Congressional Elections: Campaigning at Home and in Washington*. 4th ed. Washington, D.C.: CQ Press.

[24] Hersch, Philip L., and Gerald S. McDougall. 1994. "Campaign War Chests as a Barrier to Entry in Congressional Races." *Economic Inquiry* 32:630-641.

[25] Hogan, Robert E. 2001. "Campaign War Chests and Challenger Emergence in State Legislative Elections." *Political Research Quarterly* 54(4):815-830.

[26] Jacobson, Gary C. 1989. "Strategic Politicians and the Dynamics of U.S. House Elections, 1946-1986." *American Political Science Review* 83:773-793.

[27] Jacobson, Gary C. 2004. *The Politics of Congressional Elections*, 6th ed. New York: Longman.

[28] Jacobson, Gary C., and Samuel Kernell. 1983. *Strategy and Choice in Congressional Elections*. New Haven: Yale University Press.

[29] Kazee, Thomas A. 1980. "The Decision to Run for the U.S. Congress: Challenger Attitudes in the 1970s." *Legislative Studies Quarterly* 5:79-100.

[30] Kazee, Thomas A. 1983. "The Deterrent Effect of Incumbency on Recruiting Challengers in U.S. House Elections." *Legislative Studies Quarterly* 8:469-480.

[31] Kazee, Thomas A., ed. 1994. *Who Runs for Congress?: Ambition, Context and Candidate Emergence*. Washington, D.C.: Congressional Quarterly Press.

[32] Krasno, Jonathan S., and Donald Philip Green. 1988. "Preempting Quality Challengers in House Elections." *Journal of Politics* 50:920-936.

[33] Kreps, David M., and Robert Wilson. 1982. "Sequential Equilibrium." *Econometrica* 50:863-894.

[34] Mayhew, David R. 1974. *Congress: The Electoral Connection*. New Haven: Yale University Press.

[35] Milgrom, Paul, and Chris Shannon. 1994. "Monotone Comparative Statics." *Econometrica* 62:157-180.

[36] Milyo, Jeffrey. 1998. *The Electoral Effects of Campaign Spending in House Elections*. Los Angeles, California: Citizens' Research Foundation.

[37] Milyo, Jeffrey. 2001. "What Do Candidates Maximize (and Why Should Anyone Care)?" *Public Choice* (forthcoming).

[38] Osborne, Martin J., and Al Slivinski. 1996. "A Model of Political Competition with Citizen Candidates." *Quarterly Journal of Economics* 111:65-96.

[39] Reed, W. Robert. 1994. "A Retrospective Voting Model with Heterogeneous Politicians." *Economics and Politics* 6:39-58.

[40] Rogoff, Kenneth. 1990. "Equilibrium Political Budget Cycles." *American Economic Review* 80:21-36.

[41] Rogoff, Kenneth, and Anne Sibert. 1988. "Elections and Macroeconomic Policy Cycles." *Review of Economic Studies* 55:1-16.

[42] Squire, Peverill. 1991. "Preemptive Fund-raising and Challenger Profile in Senate Elections." *Journal of Politics* 53:1150-1164.

[43] Stone, Walter J., and L. Sandy Maisel. 2003. "The Not-So-Simple Calculus of Winning: Potential U.S. House Candidates' Nomination and General Election Prospects." *Journal of Politics* 65(4):951-977.